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**FERMION MASSES IN SUPERSTRING THEORY\***

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We give a model-independent discussion of fermion masses in four-dimensional heterotic superstring theories. We discuss the tree level contributions and quantum corrections, including one-loop threshold effects and masses generated as a result of non-perturbative supersymmetry breaking. We also point out that superstring models give rise to a generic  $\mu$ -term in the effective low energy Lagrangian.

**1. OVERVIEW**

The basic assumption of superstring phenomenology is that all light particles originate from superstring excitations that are massless at the superstring unification scale. The hierarchy between this high energy scale and small masses is created by the vacuum expectations of Higgs fields and by the supersymmetry breaking scale. Quarks and leptons acquire masses as a result of direct Yukawa couplings to Higgs scalars. The masses of squarks and sleptons receive contributions from the so-called soft terms generated by supersymmetry breaking. What is less known, and will be discussed later in this review, is that once local supersymmetry is broken, additional supersymmetric mass terms can also be generated for bosons and fermions, including the two Higgsinos of the minimal supersymmetric standard model (MSSM) and other fermions.

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According to yet another standard lore of superstring phenomenology, supersymmetry breaking occurs as a result of non-perturbative effects in some “hidden sectors” which couple to the “observable” world with gravitational-strength, non-renormalizable interactions only; the soft masses are then of order of the gravitino mass,  $m_{3/2} \propto M_{\text{HID}}^3 / M_{\text{PLANCK}}^2$ . In order for these masses to be of order 1 TeV, the scale of non-perturbative effects must be  $M_{\text{HID}} \propto 10^{14} \text{ GeV}$ . In typical superstring models, supersymmetry breaking is due to gaugino condensation of a non-abelian gauge group which becomes strong at such energy scales.

We begin by discussing fermion mass generation due to standard Yukawa interactions that originate from explicit superpotential terms

$$W = W_{ijk} Q^i Q^j h^k + s_{kl} h^k h^l + \dots, \quad (1)$$

where  $Q_{i,j}$  represent quarks or leptons, and  $h^{k,l}$  the Higgs superfields. The “Yukawa couplings”  $W_{ijk}$  are gauge singlet superfield functions. They may depend on the moduli and other fields whose vacuum expectation values (VEVs) are completely arbitrary to all orders in string perturbation theory. On the other hand, since Higgs particles are assumed to be massless at the string level, the functions  $s_{kl}$  must have zero VEVs at the string unification scale.

When computing the quark and lepton masses, special care must be taken with the wave function normalization factors. In fact, the Kähler potential can be expanded in powers of matter fields:

$$K = G + Z_{i\bar{i}} Q^i \bar{Q}^{\bar{i}} + Z_{k\bar{k}} h^k \bar{h}^{\bar{k}} + (H_{kl} h^k h^l + c.c.) + \dots \quad (2)$$

The matter-independent function  $G$  corresponds to the Kähler potential for the moduli and other gauge singlet fields. The wave function normalization factors for the matter fields are determined by the functions  $Z$ . The fermion mass matrix is

$$M_{ij} = \lambda_{ijk} \langle h^k \rangle \quad (3)$$

where the physical Yukawa couplings

$$\lambda_{ijk} = e^{G/2} (Z_{i\bar{i}} Z_{j\bar{j}} Z_{k\bar{k}})^{-1/2} W_{ijk} \quad (4)$$

and  $\langle h^k \rangle$  are the VEVs of canonically normalized Higgs fields.

The problem of computing quark and lepton masses in a given superstring model consists of two parts. The first part is to compute the functions  $G$ ,  $W_{ijk}$  and  $Z$  that enter into the physical Yukawa couplings. The most important problem at this point is to determine how these functions depend on the moduli and other gauge singlet fields with flat potentials, since VEVs of all these fields are *a priori* unknown. After solving this problem, what is generally called determination of the moduli-dependence of physical couplings, one should compute all relevant VEVs. Whereas the first part is of a purely kinematical nature, the second part involves some real superstring dynamics, like gaugino condensation and other non-perturbative phenomena.

The strategy followed in order to compute the Kähler potential and the superpotential in the effective supergravity theory of massless string excitations, is to consider the appropriate scattering amplitudes. In this way, the tree level quantities and one-loop corrections can be determined, as discussed later in this review. The tree-level superpotential (1) does not receive loop corrections, in agreement with the standard non-renormalization theorems. On the other hand, the loop expansion of the Kähler potential (2) takes the form:

$$\begin{aligned} G &= -\ln(S + \bar{S}) + G^{(0)} + \frac{2}{S + \bar{S}} G^{(1)} + \dots \\ Z &= Z^{(0)} + \frac{2}{S + \bar{S}} Z^{(1)} + \dots \\ H &= H^{(0)} + \frac{2}{S + \bar{S}} H^{(1)} + \dots \end{aligned} \quad (5)$$

Here,  $S$  is the dilaton superfield which contains the dilaton as the real part of its scalar component and the universal axion as the imaginary part.<sup>‡</sup> The dilaton VEV determines the four-dimensional string coupling constant  $g$ :  $\text{Re}\langle S \rangle = 1/g^2$

Note that the function  $H$ , which mixes Higgs particles in the Kähler potential (2), does not enter in the fermion mass formula (3). However, once local supersymmetry is broken by non-vanishing VEVs of some auxiliary fields,  $\langle F^\alpha \rangle \propto m_{3/2}$ , this mixing gives rise to Higgsino masses

$$M_{kl} = (Z_{k\bar{k}} Z_{l\bar{l}})^{-1/2} \mu_{kl}, \quad (6)$$

where

$$\mu_{kl} = m_{3/2} H_{kl} - \langle \bar{F}^{\bar{\alpha}} \rangle \partial_{\bar{\alpha}} H_{kl}. \quad (7)$$

These masses should be interpreted as originating from the effective low-energy superpotential term  $\mu_{kl} h^k h^l$  – the so-called  $\mu$ -term.

The  $\mu$ -term plays important role in the minimal supersymmetric standard model. MSSM requires the existence of two Higgs doublets,  $h^1$  and  $h^2$ , carrying opposite hypercharges.  $h^1$  provides with masses the down quarks and leptons, while  $h^2$  gives masses to up quarks. A superpotential term  $\mu h^1 h^2$  is necessary in the low energy Lagrangian in order to generate masses for Higgsinos and for undesirable electro-weak axions. In MSSM, one usually introduces by hand a parameter  $\mu$  of order of the weak scale, creating a hierarchy problem.

A class of solutions to this problem extends the MSSM to include light singlets  $s$  with Yukawa couplings to Higgs fields, as in the second term of eq.(1). This singlet could acquire a non-vanishing VEV at the electro-weak scale, driven by the soft supersymmetry breaking, generating an effective  $\mu$ -term. Similarly, a non-renormalizable effective superpotential term of the form  $M_{\text{PLANCK}}^{1-n} s^n h^1 h^2$  could be present, in which case  $\mu \propto M_{\text{PLANCK}}^{1-n} \langle s \rangle^n$ , with  $\langle s \rangle$  now of the order of some intermediate scale, such as

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<sup>‡</sup>This axion is dual to the two-index antisymmetric tensor.

the decay constant of an invisible axion.<sup>1</sup> Another solution is to introduce Kähler mixing of the form  $Hh^1h^2 + c.c.$ , as in the last term of eq.(2). In this case, the singlets are only gravitationally coupled and acquire in general Planck scale VEVs. A  $\mu$ -term can be then induced by local supersymmetry breaking,<sup>2</sup> see eq.(7). Superstring theory provides a natural setting for the latter mechanism<sup>3</sup> since  $H$  is generically a non-vanishing function of moduli fields, as discussed later in this review.

As it is clear from eqs.(3) and (7), the computation of fermion masses requires determination not only of the moduli and other scalar VEVs, but also of their auxiliary components.<sup>§</sup> This brings us back to the supersymmetry breaking problem. It is well known that gaugino condensation generates non-perturbative potential for the dilaton and moduli. Below, we give a simple physical explanation of the origin of such potentials.

In the effective low-energy Lagrangian, the gauge kinetic terms are of the form  $\sum_{\mathcal{G}} f_{\mathcal{G}} \mathcal{W}_{\mathcal{G}}^2$ , where  $\mathcal{W}_{\mathcal{G}}$  are the gauge field strength superfields and the functions  $f_{\mathcal{G}}$  are field-dependent gauge couplings associated with group  $\mathcal{G}$ :  $1/g_{\mathcal{G}}^2 = \langle f_{\mathcal{G}} \rangle$ . At the tree level, these functions are universal:  $f_{\mathcal{G}} = S$ . This universality is violated already at the one loop level by the threshold corrections which depend on the moduli as well as on the matter fields. As an example, consider gauginos of a pure gauge hidden sector. Non-perturbative effects give rise to a superpotential whose magnitude is determined by the gaugino condensate  $\langle \lambda\lambda \rangle \propto M_{\text{HID}}^3 \propto \exp(3/2bg_{\text{HID}}^2) M_{\text{PLANCK}}^3$ , where  $b$  is the one-loop beta function coefficient and  $g_{\text{HID}}$  is the coupling constant,  $1/g_{\text{HID}}^2 = \langle f_{\text{HID}} \rangle$ , of the hidden gauge group. The fact that a non-perturbative superpotential is generated for the dilaton and moduli is due to the dependence of the gauge coupling function  $f_{\text{HID}}$  on these fields. This superpotential can be derived rigorously from the effective Lagrangian describing non-perturbative gaugino condensation.<sup>4</sup> In some simple cases, like orbifold compactifications, explicit expressions can be obtained for the superpotential by using symmetry arguments based on large-small compactification radius duality.

The gauge coupling function  $f_{\text{HID}}$  depends also on the matter fields. In particular, it contains terms of the form  $M_{\text{PLANCK}}^{-2} \partial_k \partial_l f_{\text{HID}} h^k h^l$ , which depend on Higgs fields. In this way, gaugino condensation, which gives rise to  $\langle \mathcal{W}_{\text{HID}}^2 \rangle \propto M_{\text{HID}}^3$ , generates an additional contribution to the  $\mu$ -term, of order of  $m_{3/2}$ . In other words, an explicit, supersymmetric mass term is generated for Higgsinos by non-perturbative effects in hidden sectors.<sup>5,6</sup> This fact is illustrated later in this review on an orbifold example.

To summarize, there exist three basic sources of fermion masses in superstring theory: 1) tree-level superpotential Yukawa couplings, 2) mixed terms in the Kähler potential which give rise to masses once local supersymmetry is spontaneously broken, and 3) explicit mass terms generated by non-perturbative effects like gaugino condensation. In addition, fermion masses can be generated by some higher weight interactions which are briefly mentioned in section 2.

This review is organized as follows. In the next two sections, we discuss tree-level computations of the quantities that enter into determination of fermion masses, and

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<sup>§</sup>Gaugino masses can be then computed by using standard formulas.

illustrate them on orbifold examples. The last two sections are devoted to quantum corrections, illustrated again on similar orbifold examples.

## 2. TREE-LEVEL RESULTS

We will restrict our discussion to Calabi-Yau compactifications of the heterotic superstring, in which case the underlying internal superconformal field theory has  $N = (2, 2)$  world-sheet supersymmetry. In this case, the gauge group is  $E_6 \times E_8$  and the matter fields transform as **27** or  $\overline{\mathbf{27}}$  under  $E_6$  and they are in one-to-one correspondence with the moduli: **27**'s are related to (1,1) moduli and  $\overline{\mathbf{27}}$ 's to (1,2) moduli. In models with  $E_6$  grand unified group, all known particles are usually assigned to **27** representations, and Yukawa couplings originate from the superpotential terms of the form **27**<sup>3</sup>.  $E_6$  can be however broken to  $SO(10)$  or to another subgroup at the string scale; non-trivial Yukawa couplings can be then generated between particles contained in  $\overline{\mathbf{27}}$ 's and **27**'s. In the following discussion we will assume that Higgs fields can originate from both **27** and  $\overline{\mathbf{27}}$ . The knowledge of low-energy effective Lagrangians is then absolutely crucial in order to identify quarks, leptons and Higgs particles in this class of models.

The Kähler potential has the following power expansion in the matter fields:

$$K = G + A^\alpha A^{\bar{\alpha}} Z_{\alpha\bar{\alpha}}^{(1,1)} + B^\nu B^{\bar{\nu}} Z_{\nu\bar{\nu}}^{(1,2)} + (A^\alpha B^\nu H_{\alpha\nu} + c.c.) + \dots, \quad (8)$$

where  $A$  and  $B$  refer to **27**'s and  $\overline{\mathbf{27}}$ 's, respectively. The function  $G$  defines the moduli metric which at the tree-level is block-diagonal in (1,1) and (1,2) moduli:  $G^{(0)} = G^{(1,1)} + G^{(1,2)}$ . The moduli metrics as well as the matter metrics  $Z^{(1,1)}$  and  $Z^{(1,2)}$  and the Kähler mixing  $H$  have been studied in the literature up to one-loop level.<sup>7,8,5</sup>

At the tree-level the various quantities which determine the effective low energy  $N = 1$  supergravity are not independent. They are related because of the  $N = 2$  world-sheet supersymmetry in the right-moving (bosonic) sector of the heterotic superstring. An interesting consequence of the corresponding Ward-identities is the so-called special geometry, which relates the tree-level moduli metric to the Yukawa couplings:<sup>7,9</sup>

$$R_{a\bar{c}b\bar{d}}^{(0)} = G_{a\bar{c}}^{(0)} G_{b\bar{d}}^{(0)} + G_{a\bar{d}}^{(0)} G_{b\bar{c}}^{(0)} - e^{2G^{(0)}} W_{abe} \overline{W}_{\bar{c}\bar{d}\bar{f}} G^{(0)e\bar{f}}, \quad (9)$$

where  $R_{a\bar{c}b\bar{d}}^{(0)}$  is the Riemann tensor of the moduli Kähler geometry  $G^{(0)}$ , and the above equation holds separately for (1,1) and (1,2) moduli. Eq.(9) can be understood as a differential equation which determines the moduli metric in terms of the analytic superpotential and it can be solved in several examples.<sup>10</sup> On the other hand, the tree-level matter metrics are proportional to the moduli metrics:

$$\begin{aligned} Z_{\alpha\bar{\alpha}}^{(1,1)} &= G_{\alpha\bar{\alpha}}^{(1,1)} \exp(G^{(1,2)} - G^{(1,1)})/3, \\ Z_{\nu\bar{\nu}}^{(1,2)} &= G_{\nu\bar{\nu}}^{(1,2)} \exp(G^{(1,1)} - G^{(1,2)})/3. \end{aligned} \quad (10)$$

Another consequence of  $N = 2$  Ward-identities is that the Kähler mixing function  $H$  satisfies the differential equation:<sup>5</sup> ¶

$$\partial_{\bar{\beta}}\partial_{\bar{\mu}}H_{\alpha\nu}^{(0)} = G_{\alpha\bar{\beta}}^{(0)}G_{\nu\bar{\mu}}^{(0)} . \quad (11)$$

The above equation can be used to identify representations containing candidates for Higgs fields with non-trivial Kähler mixings that can result in a  $\mu$ -term.

There is a complication which arises in the presence of Yukawa couplings of the charged fields  $A, B$  with (non-moduli) singlets. The interactions induced by  $H_{AB}$  mix with some other interactions, which are not described by the standard two-derivative supergravity, corresponding to higher dimensional F-terms.<sup>5</sup> These new interactions lead to additional contribution to the effective  $\mu$ -term. In the globally supersymmetric limit, they have the form:

$$\int d^2\theta (\bar{D}^2 f^1)(\bar{D}^2 f^2) , \quad (12)$$

where  $f^{1,2}$  are arbitrary functions of moduli (and singlets) and  $\bar{D}^2$  is the chiral projection. These interactions also appear as basic building blocks in the holomorphic anomaly equations of topological amplitudes in the heterotic case.<sup>11</sup> We should note however that they vanish in the orbifold limit.

In the presence of higher weight interactions and singlets which couple to Higgs fields in the superpotential, the complete mass formula becomes rather complicated. Furthermore, Yukawa couplings of Higgs fields with singlets produce in general a direct superpotential mass since the singlets can acquire non-vanishing expectation values at the scale of supersymmetry breaking. In the case of compactifications which give rise to the particle content of the MSSM at low energies, there are no massless singlets coupled to Higgs particles and the above complication does not arise. In this case, the induced  $\mu$ -term depends entirely on the Kähler function  $H$  satisfying eq.(11), as well as on eventual non-perturbative superpotential generated at the supersymmetry breaking scale through the matter field dependence of threshold corrections to gauge couplings.

### 3. TREE-LEVEL ORBIFOLD EXAMPLES

Symmetric orbifolds are flat compactifications on the cotient of a six dimensional torus over a discrete subgroup of  $SU(3)$  so that one space-time supersymmetry remains unbroken. They correspond to singular points of Calabi-Yau manifolds with enhanced gauge symmetry  $U(1)^2$ , or larger. An important property of these models is space-time duality symmetry which contains a transformation exchanging large with small compactification radii.

For instance, for two compactified dimensions, there are four independent parameters corresponding to the three components of the metric  $G_{IJ}$  and one component of the antisymmetric tensor  $B_{IJ} = b\epsilon_{IJ}$ . They form two complex fields  $T = 2(\sqrt{G} + ib)$

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¶This solution may not be appropriate at enhanced symmetry points, for instance in orbifold compactifications.

and  $U = (\sqrt{G} + iG_{12})/G_{11}$  corresponding to (1,1) and (1,2) moduli, respectively. The duality symmetry in this case forms the group of  $SL(2, Z) \times SL(2, Z)/Z_2$  transformations,

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad U \rightarrow \frac{a'U - ib'}{ic'U + d'}, \quad T \leftrightarrow U, \quad (13)$$

where  $a, b, c, d$  are integers with  $ad - bc = 1$  (similarly for primed parameters). The matter fields  $\varphi$  transform under this transformations as  $SL(2, Z)$  modular forms of weight  $n_\varphi$ :

$$\varphi \rightarrow (icT + d)^{-n_\varphi} \varphi, \quad (14)$$

and similarly under  $U$  transformations. Moreover,  $SL(2, Z)$  duality induces a Kähler transformation under which the superpotential  $W$  transforms as a form of weight 1:

$$\begin{aligned} K &\rightarrow K + \ln(icT + d) + \ln(-ic\bar{T} + d), \\ W &\rightarrow (icT + d)^{-1} W. \end{aligned} \quad (15)$$

The massless states in orbifold models fall into two sectors: (a) The untwisted sector which contains the (1,1) and (1,2) moduli  $T_\alpha$  and  $U_\beta$ , in correspondence with the **27**'s  $A_\alpha$  and  $\overline{\mathbf{27}}$ 's  $B_\beta$ . Here,  $\alpha, \beta$  label the internal complex planes:  $\alpha = 1, 2, 3$  while  $\beta$  refers only to the  $Z_2$ -twisted planes, otherwise  $U$  is fixed to some background value. Also, in orbifolds with non abelian enhanced symmetries,  $T$  becomes a matrix and sums should be replaced by traces in the subsequent formulae. (b) The twisted sector which contains matter fields  $C$ 's and  $\overline{C}$ 's in correspondence with the blowing-up moduli which allow to deform orbifolds into regular Calabi-Yau manifolds.

The tree-level moduli metric is:<sup>7</sup>

$$G^{(0)} = - \sum_{\alpha=1}^3 \ln(T_\alpha + \overline{T}_\alpha) - \sum_{\beta} \ln(U_\beta + \overline{U}_\beta), \quad (16)$$

while the matter metrics are:

$$\begin{aligned} &U \text{ fixed:} \\ &Z_{AA}^{(0)} = \frac{1}{T + \overline{T}}, \\ &Z_2\text{-twisted planes:} \\ &Z_{AA}^{(0)} = Z_{BB}^{(0)} = \frac{1}{(T + \overline{T})(U + \overline{U})}, \\ &Z_{C\overline{C}}^{(0)} = \prod_{\alpha=1}^3 \frac{1}{(T_\alpha + \overline{T}_\alpha)^{n_\alpha^C}}, \end{aligned} \quad (17)$$

where  $T, U$  are the moduli associated to the matter fields  $A, B$ , and  $n_\alpha^C$  are the modular weights of  $C$ . Moreover, the Kähler mixing  $H$  is nonvanishing only when

the matter fields belong to the untwisted sector and are associated with a  $Z_2$ -twisted internal plane;<sup>5</sup> in addition, it depends on the moduli of this plane only:

$$\begin{aligned} H_{AB}^{(0)} &= \frac{1}{(T + \bar{T})(U + \bar{U})} , \\ H_{CC} &= H_{CA} = H_{CB} = 0 . \end{aligned} \quad (18)$$

Note that the above results are consistent with the  $SL(2, Z)$  large-small compactification radius  $T$ -duality (13-15) with the untwisted matter fields  $A$  and  $B$  having modular weights 1, provided the (1,2) modulus  $U$  is not inert in the presence of matter fields:

$$U \rightarrow U - \frac{ic}{icT + d} AB . \quad (19)$$

Similarly,  $T$  transforms under  $SL(2, Z)$   $U$ -duality.

Once supersymmetry is broken, the induced  $\mu$ -term, eq.(7), and a possible non-perturbative superpotential term  $W_{AB}AB$  yield the Higgsino mass:

$$m_{AB} = m_{3/2} + (T + \bar{T})F_T + (U + \bar{U})F_U + (T + \bar{T})(U + \bar{U})e^{G/2}W_{AB} , \quad (20)$$

where we used the tree-level expressions (16-18) for the moduli and matter metrics.

The most general superpotential, including non-perturbative contributions, has the following expansion in powers of matter fields:

$$W = W_0 + W_{AB}AB + \dots \quad (21)$$

The gravitino mass is then  $m_{3/2} = e^{G/2}W_0$ . It follows from (15) that  $SL(2, Z)$  invariance of the effective action under the transformations (13) and (19) requires that  $W_{AB}$  transforms as:

$$W_{AB} \rightarrow (icT + d)W_{AB} + ic \partial_U W_0 . \quad (22)$$

It is remarkable this transformation property automatically implies a non-vanishing mass term,  $W_{AB} \neq 0$ , if a moduli-dependent superpotential  $W_0$  is generated. It is easy to check that the physical masses (20) transform with unobservable phase factors under the duality transformations (13) and (19).

Finally, the non vanishing superpotential terms at the trilinear level are of the form:

$$W \sim A_1 A_2 A_3, B_1 B_2 B_3, ACC, BCC, CCC. \quad (23)$$

The corresponding physical Yukawa couplings between three untwisted or one untwisted and two twisted **27**'s are constants,

$$\lambda_{123}^{(0)} = \lambda_{ACC}^{(0)} = \lambda_{BCC}^{(0)} = g\sqrt{2} , \quad (24)$$

while  $\lambda_{CCC}^{(0)}$  are in general non trivial functions of the moduli  $T_\alpha$ .<sup>12</sup>

It follows that in the context of orbifold models with Higgs fields  $h^1$  and  $h^2$  contained in **27** and  $\bar{\mathbf{27}}$  of the untwisted sector, there is an automatic generation of a



$\mu$ -term induced by the breaking of local supersymmetry through their mixing in the Kähler potential. The resulting Higgsino mass is given in equation (20). Furthermore, if the top quark gets mass as a result of trilinear superpotential couplings, its Yukawa coupling is unified with the gauge couplings at the string scale and is given in eq.(24). This implies by the renormalization group evolution that the top is in general heavy with a mass close to the fixed point value. An interesting possibility is when the whole third generation receives mass from the trilinear superpotential. In this case, a strict prediction is obtained since all three Yukawa couplings are equal at the unification scale:  $\lambda_t = \lambda_b = \lambda_\tau = g\sqrt{2}$ . This leads to  $m_t \sim 180$  GeV together with a successful prediction for the bottom and tau masses in the region of large  $\tan\beta$  ( $\sim 50$ ).<sup>13</sup>

#### 4. ONE-LOOP RESULTS

Computing the loop corrections to superstring scattering amplitudes, one integrates not only over heavy particles, but over massless particles as well. This integration gives rise to on-shell infrared divergences, associated with the running of low energy couplings. In the analogous field-theoretical computations, such logarithmic divergences are usually regulated by going off-shell, to momentum  $p^2 \neq 0$ . It is then important to realize that in string theory, as well as in quantum field theory, the momentum-dependence of coupling constants is a purely infrared effect, and therefore the corresponding  $\beta$ -function coefficients of the  $p^2 \rightarrow 0$  divergence depend on the massless particle content only.<sup>14,15</sup>

Consider for instance the one-loop case. A generic on-shell amplitude  $\mathcal{A}$  corresponding to some physical coupling of the low-energy theory is written as an integral over the complex Teichmüller parameter  $\tau = \tau_1 + i\tau_2$  of the world-sheet torus inside its fundamental domain  $\Gamma \equiv \{|\tau_1| \leq \frac{1}{2}, |\tau| \geq 1\}$ :

$$\mathcal{A} = \int_{\Gamma} \frac{d^2\tau}{\tau_2} B(\tau, \bar{\tau}). \quad (25)$$

The presence of massless particles propagating in the loop implies that the integrand  $B$  goes to a constant  $b$  as  $\tau_2 \rightarrow \infty$ , and the integral over the Teichmüller parameter diverges in the infrared. When the logarithmic divergence is regularized and compared to the field-theoretical  $\overline{DR}$  scheme, it is converted to  $b \ln(M_{st}^2/p^2)$ , where  $M_{st} \simeq 5 \times g \times 10^{17}$  GeV is the string unification scale.  $b$  can then be identified with the corresponding field-theoretical  $\beta$ -function, while the remaining finite part of the integral yields the moduli-dependent string threshold corrections.<sup>15</sup>

$$\mathcal{A} = b \ln \frac{M_{st}^2}{\mu^2} + \int_{\Gamma} \frac{d^2\tau}{\tau_2} [B(\tau, \bar{\tau}) - b]. \quad (26)$$

In particular, the one-loop corrections to the Kähler metric  $K^{(1)}$  can be obtained by the computation of the one-loop three-point amplitude involving two complex

scalars and the antisymmetric tensor field:  $\mathcal{A}(b^{\mu\nu}\varphi(p_1)\overline{\varphi}(p_2)) \sim \varepsilon^{\mu\nu\lambda\rho}p_{1\lambda}p_{2\rho}K_{\varphi\overline{\varphi}}^{(1)}$ . The result is:<sup>8</sup>

$$K^{(1)} = \frac{i}{16(2\pi)^3} \int_{\Gamma} \frac{d^2\tau}{\tau_2^2} \overline{\eta}^{-2} \text{Tr}_R F(-1)^F, \quad (27)$$

where  $\eta$  is the Dedekind eta-function and the trace is over the Ramond sector of the internal  $N = 2$  (left-moving) superconformal field theory with  $U(1)$ -charge operator  $F$ .

Considering the Kähler metric corresponding to eq.(27), one obtains a finite correction for the moduli metric  $G^{(1)}$  (at a generic point  $T \neq U$ ), while the integral for the matter metric  $Z^{(1)}$  is infrared divergent. The infinite part can be identified with the one-loop anomalous dimensions of a generic  $N = 1$  supersymmetric field theory, in a gauge in which the superpotential remains unrenormalized. The remaining finite part gives the string threshold corrections to wave function factors.<sup>8</sup> These corrections determine the boundary conditions for the physical Yukawa couplings  $\lambda_{ijk}$  at the string unification scale:

$$\lambda_{ijk}(M_{st}) = \lambda_{ijk}^{(0)} [1 + g^2 (Y_i + Y_j + Y_k)]^{-1/2}, \quad (28)$$

where  $Y_i$  is defined as the finite part of  $Z_{i\bar{i}}^{(1)}/Z_{i\bar{i}}^{(0)}$ . The one-loop corrections to the Kähler mixing  $H_{AB}$  can be obtained similarly by expanding eq.(27) to first order in  $AB$ .

Finally, we discuss the matter field dependence of threshold corrections to gauge couplings which lead to an additional source of Higgsino mass via gaugino condensation. They can be obtained by the computation of the 4-point amplitude involving two gauge bosons and two matter fields  $A$  and  $B$ . It has been shown<sup>8</sup> that, to the leading order in matter fields, the one-loop threshold corrections  $\Delta^{1\text{-loop}}$  satisfy

$$\partial_i \partial_j \Delta^{1\text{-loop}} = \tilde{b} K_{ij}^{(0)} + K_{ij}^{(1)}, \quad (29)$$

where the indices  $\bar{i}, j$  represent fields which are neutral under the gauge group associated with  $\Delta^{1\text{-loop}}$ , and  $-\tilde{b}$  is the quadratic Casimir of the adjoint representation. This result can be anticipated on purely field-theoretical grounds:<sup>16,17</sup> the first term on the r.h.s. of (29) is related to anomalous graphs involving the coupling of the Kähler current to gauginos, whereas the second term is due to the Green-Schwarz term which contributes to both Kähler potential and gauge couplings.<sup>8</sup> It turns out that in the case of a pure gauge group with no massless matter field representations (like  $E_8$ ), equation (29) remains valid to higher orders in matter fields, as well.<sup>5</sup>

## 5. ONE-LOOP ORBIFOLD EXAMPLES

In orbifold models, the one-loop corrections to the relevant quantities which determine the effective field theory can be explicitly calculated. For instance, the moduli dependence of the one-loop threshold corrections to the wave function renormalization factors of untwisted matter fields are non vanishing only for fields associated to

a  $Z_2$ -twisted plane,  $A$  and  $B$ :<sup>8</sup>

$$\begin{aligned} Y_A = Y_B &= -\tilde{b}_A \{\Delta(T) + \Delta(U)\} + G^{(1)} \\ \Delta(T) &\equiv \ln(T + \bar{T}) |\eta(iT)|^4 \end{aligned} \quad (30)$$

Here,  $\tilde{b}_A = \hat{b}_A / \text{ind}$ , where  $\hat{b}_A$  is equal to the  $\beta$ -function coefficient of the gauge group that transforms  $A$  and  $B$  non-trivially in the corresponding  $N=2$  supersymmetric orbifold, and  $\text{ind}$  is the index of the little subgroup of the untwisted plane in the full orbifold group.<sup>18</sup> In eq.(30) we neglected additive constants which do not depend on the moduli  $T$  and  $U$ . It follows that the boundary relation (24) between the untwisted Yukawa couplings and the  $E_6$  gauge coupling at the unification scale does not receive any moduli-dependent corrections at the one-loop level:

$$\lambda_{123}(M_{st}) = g_{E_6}(M_{st}) \sqrt{2}, \quad (31)$$

up to moduli-independent constants. In eq.(31),  $g_{E_6}(M_{st})$  is the one-loop  $E_6$  gauge coupling constant at the unification scale.

The one-loop correction to the function  $H_{AB}$  is:<sup>5</sup>

$$H_{AB}^{(1)} = \tilde{b}_A \Delta_T(T) \Delta_U(U) - \frac{1}{U + \bar{U}} G_T^{(1)} - \frac{1}{T + \bar{T}} G_U^{(1)}, \quad (32)$$

where subscripts on the functions  $\Delta$  and  $G^{(1)}$  denote partial derivatives:  $\Delta_T(T) \equiv \partial_T \Delta(T)$  etc. The  $E_8$  threshold corrections, up to first order in the untwisted matter fields  $AB$ , read:

$$\begin{aligned} \frac{1}{g_{E_8}^2(M_{st})} &= \frac{1}{g^2} - \tilde{b}_{E_8} [\Delta(T) + \Delta(U)] + G^{(1)} + (H_{AB}^{(1)} AB + c.c.) \\ &+ \tilde{b}_{E_8} \left\{ \left[ \frac{1}{(T + \bar{T})(U + \bar{U})} - 4 \partial_T \ln \eta(iT) \partial_U \ln \eta(iU) \right] AB + c.c. \right\} \end{aligned} \quad (33)$$

with  $\tilde{b}_{E_8} = -30$ . Note that since  $G^{(1)}$  is invariant under the transformation (13), the above expressions are consistent with the invariance of the one loop correction to the Kähler potential,  $G^{(1)} + (H_{AB}^{(1)} AB + c.c.)$ , as well as of the gauge coupling  $g_{E_8}^2(M_{st})$ , under the full set of  $SL(2, Z)$  duality transformations (13-15) and (19).

We conclude by presenting an explicit expression for the Higgsino mass (20) in the context of gaugino condensation. A non-perturbative superpotential that depends on the dilaton in the right way, and satisfies the symmetry requirements (15), (22), up to first order in the expansion of matter fields  $AB$ , is:<sup>5</sup>

$$W = e^{S/2\tilde{b}_{E_8}} \eta^{-2}(iT) \eta^{-2}(iU) [1 - 4AB \partial_T \ln \eta(iT) \partial_U \ln \eta(iU)] \widetilde{W}, \quad (34)$$

where  $\widetilde{W}$  may depend on the moduli of the two other planes. The second term inside the bracket, which from the point of view of non-perturbative dynamics originates

from matter field-dependent threshold corrections (33), gives rise to a direct mass for Higgs particles.

Using auxiliary field equations and eq.(20), we obtain  $F_T = -m_{3/2}\Delta_T(T) + \mathcal{O}(g^2)$ ,  $F_U = -m_{3/2}\Delta_U(U) + \mathcal{O}(g^2)$ , and the Higgsino mass

$$m = -m_{3/2}(T + \overline{T})(U + \overline{U})\Delta_T(T)\Delta_U(U), \quad (35)$$

where we neglected terms of order  $\mathcal{O}(g^2)$ .

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## REFERENCES

- [1] E.J. Chun, J.E. Kim and H.P. Nilles, Nucl. Phys. B 370 (1992) 105.
- [2] G.F. Giudice and A. Masiero, Phys. Lett. B 206 (1988) 480.
- [3] V.S. Kaplunovsky and J. Louis, Phys. Lett. B 306 (1993) 269.
- [4] S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Phys. Lett. B 245 (1990) 409; A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B 245 (1990) 401; H.P. Nilles and M. Olechowski, Phys. Lett. B 248 (1990) 268; P. Binétruy and M.K. Gaillard, Phys. Lett. B 253 (1991) 119.
- [5] I. Antoniadis, E. Gava and K.S. Narain and T.R. Taylor, preprint hep-th 9405024.
- [6] See also: J.A. Casas and C. Muñoz, Phys. Lett. B 306 (1993) 288.
- [7] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B 329 (1990) 27.
- [8] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, Nucl. Phys. B 407 (1993) 706.
- [9] S. Ferrara, Proceedings of STRINGS 1992, Roma, September 1992 (World Scientific P. C.); P. Candelas, Proceedings of 26th Erice Workshop From Superstrings to Supergravity, December 1992 (World Scientific P. C.); and references therein.
- [10] P. Candelas, X.C. de la Ossa, P.S. Green and L. Parkes, in *Proceedings of the Second International Symposium on Particles, Strings and Cosmology*, P. Nath and S. Reucroft eds. (World Scientific, 1992), p. 667.
- [11] I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor and C. Vafa, in preparation.
- [12] S. Hamidi and C. Vafa, Nucl. Phys. B 279 (1987) 465; L. Dixon, D. Friedan, E. Martinec, S. Shenker, Nucl. Phys. B 282 (1987) 13.
- [13] W.A. Bardeen, M. Carena, S. Pokorski and C. Wagner, Phys. Lett. B 320 (1994) 110; M. Carena, M. Olechowski, S. Pokorski and C. Wagner, preprint hep-ph 9402253; S. Pokorski, preprint hep-ph 9407276.

- [14] T.R. Taylor and G. Veneziano, Phys. Lett. B 212 (1988) 147.
- [15] V.S. Kaplunovsky, Nucl. Phys. B 307 (1988) 145 and Erratum, preprint hep-th 9205070.
- [16] G.L. Cardoso and B.A. Ovrut, Nucl. Phys. B 369 (1992) 351; J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B 372 (1992) 145.
- [17] I. Antoniadis, K.S. Narain and T.R. Taylor, Phys. Lett. B 267 (1991) 37; J. Louis, in *Proceedings of the Second International Symposium on Particles, Strings and Cosmology*, P. Nath and S. Reucroft eds. (World Scientific, 1992), p. 751.
- [18] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B 355 (1991) 649.